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Stochastic Models for Precipitation

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STOCHASTIC MODELS FOR PRECIPITATION

by

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PREFACE

This report is Part II of the technical completion report for the research project entitled "Stochastic and Statistical Models for Precipitation". The project was supported by funds provided by the United States Department of Interior to the University of Kentucky Water Resources Institute as authorized by the Water Resources Act of 1964, Public Law 88-379, as Office of Water Resources Research Project No. A-055-KY.

ABSTRACT

In this project a stochastic model, using Semi-Markov Processes, was developed to simulate daily rainfall patterns in Kentucky. This model contains many of the currently used models as special cases and is applicable at any station in Kentucky as well as elsewhere.

For use in Kentucky an 8 state Semi-Markov Process is developed and the parameters of the model are determined from historical rainfall data. The model is tested at 4 different stations in Kentucky and the simulated and actual rainfall processes are found to be in good agreement. Finally some long run probabilities are calculated as well as mean return times. An appendix outlining some basic properties of the Semi-Markov Process is also included.

DESCRIPTORS

Precipitation, Semi-Markov Model.

IDENTIFIERS

Semi-Markov processes.

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1. INTRODUCTION

Many different models have been proposed for a study of the precipitation process. A Markov Chain approach to the problem was suggested by Gabriel and Neuman (1957, 1962). Recently, using this idea Allen and Haan (1975) did a detailed study of daily rainfall in Kentucky. Corvelli (1972) used a continuous time Markov process. Other authors have suggested the use of other models which are, in fact, special non-Markovian processes, see for instance Bryson (1968), and Grace and Eagleson (1966).

In this investigation we have modeled the rainfall process using a Semi-Markov chain. In appendix 1 we give a brief survey of the properties of Semi-Markov chains (S.M.C.). By choosing special cases of this model all of the previously mentioned models can be obtained.

Since we are specifically interested in Kentucky rainfall problems, the parameters of the model were estimated using data available for Ashland, Lexington, Louisville and Paducah. The validity of the model was tested via simulation and the results are presented later in this report. In order to reflect the seasonal variations in rainfall patterns, three different sets of parameters were used for the periods August 1-October 30, May 1-July 31, November 1-April 30.

Using known results from the theory of Semi-Markov processes we are able to obtain long run state probabilities and mean interstate return times. In Part I of this report, extreme value distributions are used to study the maximum daily rainfall during any year. We observe here that the same extreme value distributions also arise via a Semi-Markov model. In this way both parts of the report can be tied together.

The daily rainfall data for the four stations studied, Ashland, Lexington, Louisville and Paducah was obtained from tapes made by the division of Water, Kentucky Department of Natural Resources. For Ashland the daily rainfall data is available for a 40 year period and for the other stations, it is available for a 24 year period. There was sufficient data available so that no difficulties arose in parameter estimation.

2. The Semi-Markov Chain Model.

Daily rainfall in Kentucky is recorded to the nearest hundred of an inch. Our first task was to group the different rainfall levels into 8 different states. Here we used the same states as used by Allen and Haan, except that we split their last state in two parts. The states used were

State Number	Class Limits (hundredths of an inch)
0	0
1	1-2
2	3-6
3	7-14
4	15-30
5	31-62
6	63-126
7	127-9999.

Let now $\{J_n, n \geq 0\}$ denote successive states visited by the daily rainfall and $\{T_n, n \geq 0\}$ denote the times between state changes.

Our main assumpton is that the pair $\{J_n, T_n, n \geq 0\}$ forms a Semi-Markov chain. That is, we assume,

$$P[J_{n+1}=j,\ T_{n+1} - T_n \leq t | J_0;\ J_1, T_1, \ldots, J_n=i,\ T_n]$$

$$= P[J_{n+1}=j,\ T_{n+1} - T_n \leq t | J_n=i]. \qquad 0 \leq (i,j) \leq 7.$$

We let,

$$A_{ij}(t) = P[J_{n+1}=j,\ T_{n+1} - T_n \leq t | J_n=i]$$

and call the matrix A(t) with (i,j)th element $A_{ij}(t)$ the <u>Semi-Markov matrix</u> associated with the process. In addition and as pointed out in Pyke and Shaufele (1964) without loss of generality we assume that

$$A_{ij}(t) = P_{ij}\ F_i(t),\ 0 \leq (i,j) \leq 7,\ P_{ii}=0,\ i=0,\ldots,7.$$

where $\underline{P} = (P_{ij})$ is the transition matrix of the embedded Markov chain $\{J_n, n \geq 0\}$ and F_i is the distribution function of the time spent in state i before leaving to enter another state. It is interesting to note that if F_i is the distribution function corresponding to a geometric random variable then the Semi-Markov Chain reduces to the well known Markov Chain case. The main contribution made by the use of a Semi-Markov model is the flexibility in choice of F_i; these may be chosen in accordance with the observed data. In addition, in a Markov chain there is a one step dependency only whereas in a S.M.C. the departure probabilities depend upon the entire length of stay in a state.

In order to incorporate seasonal variation into our model, three different Semi-Markov matrices were used for the periods August 1 - October 30, November 1 - April 30 and May 1 - July 31. In a second model the same Semi-Markov was used for the entire year.

3. Parameter Estimation.

In order to utilize the above model it is necessary to have estimators of the matrix $\underline{P}$ and the waiting time distributions $F_i(t)$. To this end let n_i be the number of times in the data history that the process was in state i and let n_{ij} denote the number of jumps from state i to state j. Then, as is well known, the maximum likelihood estimator of P_{ij} is given by,

$$\hat{P}_{ij} = \frac{n_{ij}}{n_i}, \qquad 0 \leq (i,j) \leq 7, \qquad \hat{P}_{ii} = 0, \qquad i=0,\ldots,7.$$

See Anderson and Goodman (1957). Consequently our next problem was to determine $\hat{P}_{ij}$ for each of the 4 stations and for each of the above two models. These estimates are presented in the tables in Appendix 1.

Moore and Pyke (1968) discuss the problem of estimating the transition distributions of a Semi-Markov process. They suggest the estimator $\hat{P}_{ij}(t)\ \hat{F}_i(t)$ where $(0,t]$ is the time interval over which data is available, $\hat{P}_{ij}(t)$ is the Anderson and Goodman estimator given above and $\hat{F}_i(t)$ is the empirical distribution function determined from the sample. These estimators are shown to be uniformly strongly consistent as $t\to\infty$. We use the Moore and Pyke estimator in this study. In using these estimators some care must be taken as to the availability of enough data. In looking at the estimators which we obtained and also at the stationary probabilities we find that with the exception of state 0 the other states are occupied approximately 5% of the time each. In addition jumps from any state to a state other than 0 occur again approximately 5% of the time. Since it seems desirable to have at least 20 observations for each $\hat{P}_{ij}$ term this implies that we need at least 20 years data. Indeed, the 24 years which we had available is probably the smallest number of years data which should be used. For a smaller number of years it would be advisable to use fewer states resulting in a trade-off between model detail and ability to estimate parameters.

A further remark is in order. We have used the empirical distribution function to estimate length of stay in any state. If it is desired any distribution function which seems appropriate may be used. However, before using a given distribution function a goodness of fit test should be performed. In our initial investigation we determined that a geometric distribution did not provide a good fit to our data thus indicating that a Markov chain was not the model to use.

4. Simulation Experience.

In order to test the validity of our two models we did 10 simulations of daily rainfall at each of the four stations. Each simulation run for Ashland was over a 40 year period while a 24 year period was used for each of the remaining three stations. In each simulation the initial state was chosen from the data (January 1, 1932 for Ashland, June 1, 1948 for each of the others). A random variable signifying the length of stay in the initial state was then generated using the appropriate $\hat{F}_i(x)$ distribution and the next state to be entered was generated using the $\hat{P}_{ij}$ elements. This procedure was continued until the run was complete. After each new state was chosen, the actual rainfall level was taken to be the mean rainfall in that state for the actual data.. Allen and Haan (1975) and Carey and Haan (1976) suggest that a gamma distribution would be a more appropriate choice. Since our main goal was to introduce the idea of using a Semi-Markov model to generate a sequence of states we choose the simpler approach. At each station we carried out the simulation using the "3 seasons" model and the "whole year" model. The grand summaries of the results of these simulations are given in Appendix 1.

Because of the lack of independence between successive daily rainfalls exact statistical tests to compare simulated and actual data are unavailable. However we have computed some descriptive statistics.

In tables A.5, A.6, B.5, B.6, C.5, C.6 and D.5, D.6 we have listed the actual and simulated monthly means and variances. The agreement is good; however whether or not the fit is good enough will depend upon the use to which the model is put. In addition we have also compared mean annual rainfalls; here the difference between simulated and actual values is in the neighborhood of 1%. Again this shows a good fit by the model.

In order to gain some insight into the accuracy of the fit on a daily level we proceeded as follows. Let $X_1,\ldots,X_N$ denote the actual seuqence of rainfall levels at a particular station and $X_1^*,\ldots,X_N^*$ the actual sequence of rainfall states, where N denotes the total number of days involved. Let $Y_1,\ldots,Y_N$ and $Y_1^*,\ldots,Y_N^*$ be the corresponding values for the simulated data. Then,

(a.) Ave. absolute difference between simulated and actual states

$$= \frac{1}{N} \sum_{i=1}^{N} |X_i^* - Y_i^*|$$

(b.) Ave. absolute difference between simulated and actual rainfall

levels $= \frac{1}{N} \sum_{i=1}^{N} |X_i - Y_i|$

These numbers are also reported in the above tables. Unfortunately, we do not know the distributions of the above statistics. An examination of the tables shows that for (b) the values are around 0.2". Again, how acceptable this fit is will depend upon the purposes to which the model is being put. One point which was notable was the overall lack of difference between the "3 seasons" and the "whole year" models. In general we would conclude that, on the basis of the above comparisons, our models perform reasonably well.

5. Stationary State Probabilities.

In tables A.8. B.8, C.8 and D.8 we give the long run or stationary state probabilities based on the "whole year" model. The procedure for computing these probabilities is given in Appendix 2. These figures represent probabilities when the effect of the initial choice of starting state has worn off. In examining these tables we find that in both Ashland and Lexington the long run probability of a dry day is 64%. We can also interpret this to mean that in the long run, approximately 64% of the time these two stations will be in state 0. The figures for Louisville and Paducah are 62% and 69% respectively. In order to generate simulated data over some fixed future period, the initial state should be chosen according to these stationary state probabilities with the remainder of the simulation being done as indicated earlier.

Another use to which we can put the stationary state probabilities is the calculation of the mean time between enteries to a given state. For each state i, let μ_{ii} denote the mean return time to the state given that it has just been vacated. We give in Appendix No. 2 a formula for calculating μ_{ii} for each state i. As an example, for Louisville we see that the value of μ_{77} is 59 days (more exactly, 58.76 days). We interpret this to mean that between the end of one period of days wherein the daily rainfall level has exceeded 1.26" of rain and the beginning of the next such period we have 59 days on the average. Similar figures can be calculated for each of the other 3 stations.

We have also given in our tables the stationary state probabilities for the embedded markov chain. These two sets of probabilities differ since the S.M.C. probabilities also take into account the mean in state waiting times.

In part I of this report we investigated the distribution of the extreme daily rainfall during the year. To briefly recall this discussion let X_i denote the amount of rainfall on day i. Let $Y_1 = \max_{1 \leq i \leq 365} X_i$ then the distribution function of Y can be approximated using one of the extreme value distributions. This same discussion carries over to the present case using the results of Neuts and Resnick (1971).

6. Summary.

The Semi-Markov Chain model of section 2 seems to be a good model for the daily rainfall process in Kentucky. In addition, there is enough flexibility in the model that its use should not be restricted only to Kentucky. It provides a natural extension and a general setting for many other models which have already appeared in the literature.

As demonstrated using data at 4 stations in Kentucky there are no major difficulties in estimating the parameters of the model. Also, there is not much predicitive value lost in using a "whole year" model rather than a "3 seasons" model.

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TABLE A1: P and f matrices for Ashland -- (May - July)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.085	0.132	0.181	0.216	0.214	0.129	0.042
0.541	0.0	0.115	0.090	0.098	0.098	0.049	0.008
0.584	0.062	0.0	0.062	0.101	0.096	0.062	0.034
0.551	0.047	0.051	0.0	0.126	0.126	0.061	0.037
0.598	0.035	0.055	0.071	0.0	0.138	0.083	0.020
0.546	0.062	0.106	0.103	0.103	0.0	0.059	0.022
0.465	0.057	0.088	0.119	0.088	0.164	0.0	0.019
0.534	0.121	0.069	0.017	0.086	0.138	0.034	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.281	0.951	0.944	0.883	0.890	0.887	0.906	0.983
2	0.219	0.049	0.051	0.107	0.098	0.095	0.082	0.017
3	0.153	0.0	0.0	0.0	0.008	0.015	0.013	0.0
4	0.096	0.0	0.006	0.009	0.004	0.004	0.0	0.0
5	0.063	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.037	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.045	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.030	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.024	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.017	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.016	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE A2: P and f matrices for Ashland -- (August - October)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.120	0.192	0.151	0.178	0.213	0.109	0.039
0.667	0.0	0.053	0.088	0.079	0.035	0.053	0.026
0.612	0.041	0.0	0.082	0.065	0.094	0.076	0.029
0.686	0.057	0.043	0.0	0.071	0.086	0.036	0.021
0.624	0.071	0.106	0.059	0.0	0.088	0.035	0.018
0.623	0.044	0.104	0.060	0.077	0.0	0.066	0.027
0.505	0.084	0.084	0.047	0.150	0.103	0.0	0.028
0.409	0.045	0.091	0.091	0.205	0.091	0.068	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.190	0.947	0.942	0.943	0.900	0.902	0.916	1.000
2	0.181	0.044	0.053	0.057	0.100	0.098	0.075	0.0
3	0.111	0.009	0.006	0.0	0.0	0.0	0.009	0.0
4	0.109	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.104	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.063	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.070	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.028	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.046	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.018	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.012	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE A2: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
19	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE A3: P and f matrices for Ashland -- (November - April)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.135	0.176	0.193	0.196	0.191	0.092	0.017
0.665	0.0	0.087	0.061	0.069	0.072	0.033	0.013
0.598	0.066	0.0	0.081	0.100	0.088	0.058	0.009
0.598	0.082	0.080	0.0	0.074	0.090	0.053	0.023
0.562	0.100	0.076	0.102	0.0	0.100	0.052	0.010
0.474	0.090	0.101	0.113	0.123	0.0	0.072	0.027
0.447	0.084	0.128	0.084	0.114	0.103	0.0	0.040
0.324	0.041	0.135	0.122	0.122	0.149	0.108	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.291	0.913	0.919	0.910	0.867	0.895	0.927	0.959
2	0.254	0.077	0.077	0.084	0.118	0.105	0.070	0.041
3	0.155	0.008	0.004	0.006	0.016	0.0	0.004	0.0
4	0.097	0.003	0.0	0.0	0.0	0.0	0.0	0.0
5	0.073	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE A3: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
6	0.045	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.032	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.017	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE A4: P and f matrices for Ashland -- (All Year)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.118	0.169	0.183	0.197	0.201	0.105	0.027
0.642	0.0	0.083	0.072	0.077	0.070	0.042	0.014
0.597	0.060	0.0	0.077	0.097	0.089	0.061	0.020
0.595	0.069	0.067	0.0	0.090	0.100	0.054	0.025
0.583	0.078	0.076	0.082	0.0	0.110	0.058	0.014

TABLE A4: Continued

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.524	0.073	0.104	0.101	0.107	0.0	0.066	0.025
0.464	0.078	0.109	0.085	0.109	0.121	0.0	0.033
0.415	0.068	0.102	0.074	0.131	0.136	0.074	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.266	0.923	0.930	0.906	0.880	0.894	0.918	0.977
2	0.230	0.069	0.065	0.088	0.108	0.101	0.074	0.023
3	0.146	0.006	0.004	0.004	0.011	0.004	0.007	0.0
4	0.100	0.002	0.001	0.002	0.001	0.001	0.0	0.0
5	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.045	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.043	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.023	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.020	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.012	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.008	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE A5: Comparison of simulated and actual rainfall averages at Ashland using the 3 seasons model.

Average Absolute Difference Between Simulated and Actual States = 1.89

Average Absolute Difference Between Simulated and Actual Rainfall = 0.186
(Value of this measure for the actual data is 0.020)

Mean of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Simulated	3.50	3.06	3.40	3.22	4.06	3.93	4.05	2.82	2.88	2.64	3.17	3.45	40.18
Actual	3.43	2.94	4.12	3.52	3.93	3.62	4.38	3.41	2.77	2.02	2.82	2.95	39.91

Standard Deviation of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Simulated	1.62	1.51	1.56	1.53	1.68	1.78	1.75	1.62	1.64	1.54	1.60	1.61
Actual	2.08	1.75	1.73	1.56	1.86	1.80	1.94	1.70	1.40	1.05	1.37	1.49

TABLE A6: Comparison of simulated and actual rainfall averages at Ashland using the "all year" model.

Average Absolute Difference Between Simulated and Actual States = 1.91

Average Absolute Difference Between Simulated and Actual Rainfall = 0.188
(Value of this measure for the actual data is 0.020)

Mean of Montly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Simulated	3.32	3.12	3.44	3.23	3.32	3.43	3.61	3.41	3.34	3.43	3.24	3.38	40.27
Actual	3.43	2.94	4.12	3.52	3.93	3.62	4.38	3.41	2.77	2.02	2.82	2.95	39.91

Standard Deviation of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Simulated	1.70	1.63	1.71	1.67	1.56	1.69	1.70	1.66	1.71	1.60	1.62	1.68
Actual	2.08	1.75	1.73	1.56	1.86	1.80	1.94	1.70	1.40	1.05	1.37	1.49

TABLE A7: Stationary state probabilities for Embedded Markov Chain -- Ashland.

State	0	1	2	3	4	5	6	7
Probability	.3602	.0821	.1072	.1101	.1215	.1237	.0707	.0231

TABLE A8: Stationary probabilities for S.M.C. at Ashland.

State	0	1	2	3	4	5	6	7
Probability	.6466	.0452	.0591	.0615	.0698	.0668	.0390	.0120

TABLE B1: P and f matrices for Lexington -- (May - July)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.134	0.132	0.154	0.196	0.176	0.147	0.060
0.528	0.0	0.066	0.104	0.075	0.142	0.085	0.0
0.689	0.058	0.0	0.087	0.049	0.058	0.058	0.0
0.597	0.037	0.067	0.0	0.104	0.060	0.097	0.037
0.557	0.087	0.060	0.087	0.0	0.067	0.087	0.054
0.510	0.084	0.063	0.105	0.084	0.0	0.112	0.042
0.539	0.063	0.055	0.094	0.094	0.141	0.0	0.016
0.313	0.063	0.063	0.104	0.208	0.146	0.104	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.321	0.953	0.951	0.948	0.946	0.930	0.906	0.979
2	0.201	0.047	0.039	0.045	0.054	0.070	0.086	0.021
3	0.152	0.0	0.010	0.007	0.0	0.0	0.008	0.0
4	0.092	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.069	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.031	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.040	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.031	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.025	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.011	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.013	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE B2: P anf f matrices for Lexington -- (August - October)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.139	0.153	0.205	0.156	0.213	0.088	0.045
0.722	0.0	0.056	0.056	0.067	0.022	0.056	0.022
0.656	0.111	0.0	0.078	0.044	0.056	0.056	0.0
0.670	0.040	0.100	0.0	0.100	0.050	0.030	0.010
0.593	0.110	0.066	0.044	0.0	0.099	0.066	0.022
0.553	0.087	0.087	0.058	0.078	0.0	0.087	0.049
0.590	0.098	0.066	0.066	0.082	0.082	0.0	0.016
0.481	0.074	0.074	0.074	0.111	0.074	0.111	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.213	0.944	0.933	0.970	0.934	0.874	0.919	0.963
2	0.139	0.056	0.067	0.030	0.066	0.126	0.081	0.037
3	0.134	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.122	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.102	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.051	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.034	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.054	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.028	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.020	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.034	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.011	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.011	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE B2: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
19	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE B3: P and f matrices for Lexington -- (November - April)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.167	0.161	0.152	0.194	0.175	0.117	0.035
0.635	0.0	0.074	0.046	0.077	0.084	0.074	0.011
0.618	0.099	0.0	0.069	0.084	0.073	0.046	0.011
0.566	0.052	0.068	0.0	0.088	0.127	0.064	0.036
0.543	0.110	0.072	0.093	0.0	0.096	0.062	0.024
0.483	0.106	0.110	0.116	0.092	0.0	0.075	0.017
0.478	0.120	0.105	0.067	0.077	0.096	0.0	0.057
0.352	0.085	0.042	0.099	0.085	0.141	0.197	0.0

TABLE B3: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.301	0.923	0.912	0.948	0.911	0.928	0.895	0.915
2	0.254	0.074	0.084	0.052	0.082	0.065	0.100	0.070
3	0.156	0.004	0.004	0.0	0.007	0.007	0.005	0.014
4	0.102	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.062	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.048	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.031	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.018	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.008	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE B4: P and f matrices for Lexington -- (All Year)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.152	0.150	0.163	0.188	0.183	0.119	0.044
0.630	0.0	0.067	0.062	0.075	0.083	0.073	0.010
0.641	0.095	0.0	0.075	0.066	0.066	0.051	0.007
0.594	0.045	0.076	0.0	0.093	0.095	0.066	0.031
0.554	0.105	0.068	0.081	0.0	0.087	0.073	0.032

TABLE B4: Continued

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.502	0.095	0.095	0.104	0.088	0.0	0.086	0.030
0.511	0.100	0.085	0.075	0.083	0.108	0.0	0.038
0.363	0.075	0.055	0.096	0.130	0.130	0.151	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.287	0.934	0.921	0.953	0.925	0.916	0.902	0.945
2	0.216	0.064	0.077	0.045	0.072	0.078	0.093	0.048
3	0.149	0.002	0.002	0.002	0.004	0.006	0.005	0.007
4	0.106	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.075	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.043	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.035	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.026	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.016	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.008	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE B4: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
24	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
31	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
33	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE B5: Comparison of simulated and actual rainfall averages at Lexington using the 3 seasons model.

Average Absolute Difference Between Simulated and Actual States = 1.92

Average Absolute Difference Between Simulated and Actual Rainfall = 0.218
(Value of this measure for the actual data is 0.029)

Mean of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Simulated	4.40	3.91	4.16	4.14	4.50	4.55	4.58	2.81	2.76	2.57	3.67	4.25	46.30
Actual	4.05	3.54	4.69	5.41	4.35	4.42	4.75	3.29	2.87	1.97	3.40	3.85	46.59

Standard Deviation of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Simulated	2.04	1.94	2.05	2.13	2.18	2.26	2.06	1.58	1.73	1.59	2.04	2.13
Actual	3.36	1.93	2.39	6.69	1.85	2.07	2.08	1.13	1.55	1.06	1.46	1.95

TABLE B6: Comparison of simulated and actual rainfall averages at Lexington using the "all year" model.

Average Absolute Difference Between Simulated and Actual States = 1.94

Average Absolute Difference Between Simulated and Actual Rainfall = 0.219
(Value of this measure for the actual data is 0.029)

Mean of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Simulated	3.89	3.61	3.68	3.92	4.19	3.95	3.89	3.91	3.80	3.92	3.81	3.84	46.41
Actual	4.05	3.54	4.69	5.41	4.35	4.42	4.75	3.29	2.87	1.97	3.40	3.85	46.59

Standard Deviation of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Simulated	1.95	1.98	2.05	2.02	2.20	2.00	2.13	2.14	1.86	1.90	2.02	2.22
Actual	3.36	1.93	2.39	6.69	1.85	2.07	2.08	1.13	1.55	1.06	1.46	1.95

TABLE B7: Stationary state probabilities for Embedded Markov Chain -- Lexington.

State	0	1	2	3	4	5	6	7
Probability	.3590	.1016	.0957	.1023	.1120	.1130	.0841	.0308

TABLE B8: Stationary Probabilities for S.M.C. at Lexington.

State	0	1	2	3	4	5	6	7
Probability	.6431	.0563	.0536	.0556	.0626	.0638	.0481	.0159

TABLE C1: P and f matrices for Louisville -- (May - July)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.132	0.132	0.203	0.184	0.179	0.122	0.048
0.552	0.0	0.124	0.057	0.105	0.095	0.029	0.038
0.592	0.041	0.0	0.061	0.092	0.112	0.082	0.020
0.581	0.093	0.054	0.0	0.085	0.101	0.062	0.023
0.571	0.090	0.038	0.068	0.0	0.098	0.098	0.038
0.533	0.088	0.066	0.066	0.117	0.0	0.102	0.029
0.571	0.067	0.067	0.095	0.067	0.114	0.0	0.019
0.450	0.075	0.050	0.100	0.050	0.075	0.200	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.284	0.924	0.949	0.953	0.880	0.869	0.924	0.950
2	0.198	0.067	0.041	0.031	0.105	0.131	0.076	0.050
3	0.136	0.010	0.010	0.016	0.015	0.0	0.0	0.0
4	0.100	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.084	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.060	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.031	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.029	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.021	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.010	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.017	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE C2: P and f matrices for Louisville -- (August - October)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.145	0.118	0.173	0.199	0.165	0.145	0.055
0.759	0.0	0.013	0.051	0.051	0.076	0.013	0.038
0.609	0.047	0.0	0.094	0.094	0.078	0.063	0.016
0.652	0.056	0.045	0.0	0.135	0.079	0.034	0.0
0.713	0.074	0.056	0.056	0.0	0.028	0.065	0.009
0.685	0.045	0.090	0.045	0.045	0.0	0.056	0.034
0.521	0.085	0.042	0.113	0.113	0.099	0.0	0.028
0.448	0.103	0.034	0.034	0.172	0.138	0.069	0.0

Waitng Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.173	0.987	0.953	0.944	0.926	0.888	0.931	0.931
2	0.173	0.013	0.047	0.056	0.074	0.112	0.069	0.069
3	0.127	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.101	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.090	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.075	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.055	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.055	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.029	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.029	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.014	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.017	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.012	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.012	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE C2: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
31	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
33	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
36	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE C3: P and f matrices for Louisville -- (November - April)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.155	0.151	0.166	0.191	0.180	0.113	0.044
0.680	0.0	0.050	0.064	0.071	0.078	0.046	0.011
0.554	0.103	0.0	0.089	0.103	0.080	0.063	0.009
0.592	0.112	0.072	0.0	0.044	0.088	0.072	0.020

TABLE C3: Continued

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.615	0.091	0.036	0.073	0.0	0.095	0.073	0.018
0.464	0.136	0.100	0.104	0.075	0.0	0.082	0.039
0.483	0.139	0.055	0.065	0.104	0.119	0.0	0.035
0.472	0.014	0.111	0.028	0.111	0.097	0.167	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.303	0.918	0.924	0.948	0.931	0.900	0.095	0.986
2	0.230	0.075	0.071	0.048	0.065	0.093	0.080	0.014
3	0.152	0.007	0.004	0.004	0.004	0.007	0.015	0.0
4	0.110	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.055	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.060	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.028	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.018	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.013	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE C4: P and f matrices for Louisville -- (All Year)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.146	0.138	0.177	0.192	0.177	0.122	0.047
0.658	0.0	0.065	0.062	0.075	0.082	0.037	0.022
0.573	0.078	0.0	0.080	0.101	0.088	0.067	0.013
0.593	0.099	0.062	0.0	0.073	0.092	0.064	0.017
0.624	0.089	0.039	0.068	0.0	0.081	0.078	0.021
0.517	0.107	0.093	0.083	0.081	0.0	0.083	0.036
0.513	0.108	0.058	0.082	0.095	0.114	0.0	0.029
0.461	0.050	0.078	0.050	0.106	0.092	0.163	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.267	0.931	0.935	0.946	0.917	0.887	0.915	0.965
2	0.210	0.062	0.060	0.047	0.078	0.109	0.077	0.035
3	0.145	0.006	0.005	0.006	0.006	0.004	0.008	0.0
4	0.105	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.073	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.060	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.036	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.029	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.017	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.015	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.011	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE C4: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
31	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
33	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	0.0	0.0	0.0	0.0	0 0	0.0	0.0	0.0
35	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE C5: Comparison of simulated and actual rainfall averages at Louisville using the 3 seasons model.

Average Absolute Difference Between Simulated and Actual States = 1.86

Average Absolute Difference Between Simulated and Actual Rainfall = 0.203
(Value of this measure for the actual data is 0.023)

Mean of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Simulated	3.87	3.56	3.87	3.78	3.95	3.78	4.11	2.92	2.76	2.83	3.47	3.95	42.89
Actual	3.72	3.57	4.66	4.12	4.36	3.70	3.88	2.91	3.04	2.33	3.39	3.48	43.15

Standard Deviation of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Simulated	1.82	1.89	1.92	1.92	2.15	1.86	1.89	1.60	1.70	1.62	1.85	2.04
Actual	2.41	2.09	2.96	2.53	1.97	2.28	1.85	1.67	1.76	1.24	2.03	1.49

TABLE C6: Comparison of simulated and actual rainfall averages at Louisville using the "all year" model.

Average Absolute Difference Between Simulated and Actual States = 1.88

Average Absolute Difference Between Simulated and Actual Rainfall = 0.204
(Value of this measure for the actual data is 0.023)

Mean of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Simulated	3.67	3.31	3.64	3.56	3.58	3.69	3.65	3.65	3.59	3.64	3.53	3.47	42.99
Actual	3.72	3.57	4.66	4.12	4.36	3.70	3.88	2.91	3.04	2.33	3.39	3.48	43.15

Standard Deviation of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Simulated	1.90	1.92	1.82	1.88	1.68	1.80	1.86	1.83	2.04	1.87	1.87	1.84
Actual	2.41	2.09	2.96	2.53	1.97	2.28	1.85	1.67	1.76	1.24	2.03	1.49

TABLE C7: Stationary state probabilities for Embedded Markov Chain -- Louisville.

State	0	1	2	3	4	5	6	7
Probability	.3650	.1031	.0856	.1035	.1144	.1120	.0838	.0313

TABLE C8: Stationary Probabilities for S.M.C. at Louisville.

State	0	1	2	3	4	5	6	7
Probability	.6279	.0599	.0497	.0595	.0677	.0681	.0495	.0177

TABLE D1: P and f matrices for Paducah -- (May - July)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.071	0.128	0.148	0.210	0.199	0.168	0.077
0.585	0.0	0.038	0.132	0.038	0.113	0.057	0.038
0.627	0.060	0.0	0.075	0.075	0.090	0.015	0.060
0.527	0.054	0.022	0.0	0.129	0.129	0.097	0.043
0.591	0.047	0.047	0.110	0.0	0.102	0.079	0.024
0.603	0.026	0.026	0.060	0.129	0.0	0.086	0.069
0.602	0.087	0.058	0.049	0.117	0.058	0.0	0.029
0.451	0.020	0.059	0.059	0.137	0.059	0.216	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.229	0.981	0.970	0.925	0.929	0.862	0.913	0.961
2	0.187	0.019	0.030	0.075	0.071	0.121	0.078	0.039
3	0.136	0.0	0.9	0.0	0.0	0.017	0.010	0.0
4	0.102	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.076	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.076	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.045	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.011	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.017	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.034	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.023	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.028	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.014	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE D1: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
18	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.006	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE D2: P and f matrices for Paducah -- (August - October)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.097	0.141	0.148	0.141	0.225	0.178	0.070
0.640	0.0	0.020	0.100	0.100	0.060	0.020	0.060
0.714	0.043	0.0	0.029	0.0	0.086	0.043	0.086
0.466	0.041	0.068	0.0	0.110	0.123	0.123	0.068
0.704	0.056	0.056	0.085	0.0	0.014	0.028	0.056
0.594	0.021	0.135	0.083	0.073	0.0	0.063	0.031
0.658	0.066	0.053	0.053	0.039	0.092	0.0	0.039
0.545	0.091	0.023	0.091	0.136	0.068	0.045	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.148	1.000	0.943	0.945	0.958	0.948	0.947	0.978
2	0.138	0.0	0.043	0.055	0.028	0.052	0.053	0.022
3	0.141	0.0	0.014	0.0	0.014	0.0	0.0	0.0
4	0.107	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.104	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.047	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.054	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.067	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.044	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE D2: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
10	0.013	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.030	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.020	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.013	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.010	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.017	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.010	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
31	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
33	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE D3: P and f matrices for Paducah -- (November - April)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.115	0.142	0.183	0.187	0.191	0.125	0.056
0.642	0.0	0.068	0.074	0.074	0.062	0.056	0.025
0.673	0.015	0.0	0.069	0.054	0.084	0.045	0.059
0.543	0.076	0.067	0.0	0.081	0.094	0.090	0.049
0.518	0.073	0.064	0.077	0.0	0.136	0.095	0.036
0.587	0.071	0.126	0.083	0.031	0.0	0.055	0.047
0.578	0.056	0.078	0.061	0.083	0.067	0.0	0.078
0.337	0.096	0.077	0.077	0.125	0.173	0.115	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.249	0.920	0.941	0.915	0.918	0.953	0.922	0.933
2	0.211	0.074	0.045	0.085	0.077	0.047	0.072	0.067
3	0.156	0.006	0.015	0.0	0.005	0.0	0.006	0.0
4	0.090	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.084	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.065	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.037	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.031	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.017	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.009	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.010	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.008	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE D3: Continued

Waiting Time Distrubiton for Each State:

Days in State	0	1	2	3	4	5	6	7
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE D4: P and f matrices for Paducah -- (All Year)

Transition Probability Matrix:

0	1	2	3	4	5	6	7
0.0	0.100	0.138	0.169	0.184	0.199	0.146	0.064
0.626	0.0	0.053	0.091	0.072	0.072	0.053	0.034
0.667	0.029	0.0	0.062	0.050	0.088	0.038	0.065
0.524	0.064	0.059	0.0	0.098	0.108	0.095	0.051
0.568	0.062	0.060	0.086	0.0	0.107	0.081	0.036
0.591	0.049	0.101	0.077	0.065	0.0	0.065	0.052
0.595	0.070	0.067	0.056	0.087	0.070	0.0	0.056
0.410	0.075	0.060	0.075	0.130	0.125	0.125	0.0

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
1	0.221	0.947	0.947	0.923	0.931	0.927	0.922	0.950
2	0.190	0.049	0.041	0.077	0.064	0.069	0.073	0.050
3	0.146	0.004	0.012	0.0	0.005	0.004	0.006	0.0
4	0.098	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.087	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.063	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.043	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.033	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.023	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE D4: Continued

Waiting Time Distribution for Each State:

Days in State	0	1	2	3	4	5	6	7
10	0.016	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.016	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.016	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.013	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.008	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.007	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.004	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.002	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
31	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
33	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE D5: Comparison of simulated and actual rainfall averages at Paducah using the 3 seasons model.

Average Absolute Difference Between Simulated and Actual States = 1.84

Average Absolute Difference Between Simulated and Actual Rainfall = 0.229
(Value of this measure for the actual data is 0.028)

Mean of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Simulated	4.46	3.79	4.30	4.03	4.61	4.32	4.29	3.51	3.17	3.25	4.00	4.15	47.90
Actual	4.25	3.68	4.82	4.61	4.79	4.25	3.88	3.15	3.46	2.78	3.58	4.17	47.43

Standard Deviation of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Simulated	2.27	2.11	2.24	2.15	2.46	2.23	2.23	1.97	1.87	2.01	2.21	2.14
Actual	3.15	1.89	3.13	2.30	2.38	2.50	2.78	1.54	1.97	1.79	2.79	1.93

TABLE D6: Comparison of simulated and actual rainfall averages at Paducah using the "All Year" model.

Average Absolute Difference Between Simulated and Actual States = 1.85

Average Absolute Difference Between Simulated and Actual Rainfall = 0.230
(Value of this measure for the actual data is 0.028)

Mean of Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
Simulated	4.14	3.51	4.14	4.16	4.11	3.79	3.79	4.00	4.05	4.05	3.98	3.93	47.66
Actual	4.25	3.68	4.82	4.61	4.79	4.25	3.88	3.15	3.46	2.78	3.58	4.17	47.43

Standard Deviation of Monthly Precipitation

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Simulated	2.25	2.03	2.20	2.31	2.21	2.06	2.07	2.18	2.23	2.33	2.21	2.20
Actual	3.15	1.89	3.13	2.30	2.38	2.50	2.78	1.54	1.97	1.79	2.79	1.93

TABLE D7: Stationary State Probabilities for Embedded Markov Chain -- Paducah.

State	0	1	2	3	4	5	6	7
Probability	.3652	.0689	.0882	.1012	.1090	.1210	.0931	.0520

TABLE D8: Stationary Probabilities for S.M.C. at Paducah.

State	0	1	2	3	4	5	6	7
Probability	.6990	.0328	.0416	.0482	.0519	.0576	.0447	.0243

Appendix 2: Summary of some results concerning Semi-Markov Processes.

1. Preliminaries:

Let $\{J_n; \; n \geq 0\}$ be a sequence of random variables which can assume values in the set $\{0,1,2,\ldots,m\}$ and let $\{T_n; \; n \geq 0\}$ be non-negative random variables.

<u>Definition</u>: $\{J_n, T_n, n \geq 0\}$ is a Semi-Markov Process if for $n > 0$, $0 \leq (ij) \leq m$,

$$P\{J_{n+1} = j, \; T_{n+1} - T_n \leq t \,|\, J_0, J_1, T_1, \ldots, J_n = i, T_n\}$$

$$= \; P\{J_{n+1} = j, \; T_{n+1} - T_n \leq t \,|\, J_n = i\} \quad \text{A.S.}$$

Let $A_{ij}(t) = P[J_{n+1} = j, \; T_{n+1} - T_n \leq t \,|\, J_n = i]$ and let $A(t) = [A_{ij}(t)]$.

Let $P_{ij} = A_{ij}\;(\infty)$ and $F_{ij}(t) = \dfrac{A_{ij}(t)}{P_{ij}}$

Then it is an immediate consequence of the above definition that $\{J_n, n \geq 0\}$ is a Markov Chain with transition matrix $\underline{P} = (P_{ij})$. $F_{ij}(t)$ represents the conditional probability that a transition will occur within an amount of time t given that the process has just entered i and will next enter j. We will refer to $\{J_n, n \geq 0\}$ as the Embedded Markov Chain.

Now let $N_i(t)$ denote the number of transitions into state i in $(0,t]$ and let,

$$N(t) = \sum_{i=0}^{m} N_i(t).$$

If $Z(t)$ denotes the state of the process at time t then,

$$Z(t) = J_{N(t)} \; .$$

It is not too hard to see that a S.M.P. behaves like a Markov Chain except that the times between jumps need not be Exponential or Geometric.

2. Limiting distributions for S.M.P.

We state here the necessary limit theorems for a S.M.P. Let,

$$\mu_i = \int_0^\infty \sum_{j=0}^{m} t \; d\, F_{ij}(t); \quad i=0,\ldots,m.$$

Then μ_i is the mean time spent in state i. Also, let

$$\pi_i = \lim_{n\to\infty} P[J_n = i]$$

Then π_i represent the stationary probabilities for the Markov Chain $\{J_n, n \geq 0\}$ and, if $\pi = (\pi_0, \ldots, \pi_m)$ then π is the unique solution to the system of equations,

$$\pi = \pi P$$

$$\sum_{i=0}^{m} \pi_i = 1$$

The main theorem we need is the following.

<u>Theorem</u>: $\lim_{t\to\infty} P[Z(t) = j] = \dfrac{\pi_j \mu_j}{\sum_{i=0}^{m} \pi_i \mu_i} = P_j, \quad j=0,\ldots,m.$

These are the probabilities which we have tabulated in Table 8 of Appendix 1.

Another quantity of interest is the mean return time to a given state, say μ_{jj}. This is related to the P_j terms by,

$$\mu_{jj} = \frac{\mu_j}{P_j} \quad j=0,\ldots,m.$$

For a detailed account of Semi Markov Processes see the text by Ross (1970) or Cinlar (1975).

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